

Hypothesis Testing

In statistics, we are often concerned with claims that are made. For example, the government claims that the mean income is \$32,067, or a drug company claims that the mean incidence rate for side effects to their drug is less than or equal to 2.81 (per 100,000 people). In this section, we'll outline the traditional method of testing these claims.

**Definition:** A *hypothesis* is a claim that something is true. In statistics, there are two hypotheses with every problem. The *null hypothesis*, denoted by  $H_0$ , is the claimed value of the statistic (the mean for example). The *alternative hypothesis*, denoted by  $H_1$ , is the statement that is true whenever the null hypothesis is false.

**Example 1:** Suppose the government claims that the mean income is \$32,067. State the null hypothesis and the alternative hypothesis.

$$H_0 : \mu = 32,067$$

$$H_1 : \mu \neq 32,067$$

**Example 2:** Suppose a drug company claims that the mean incidence rate for side effects to their drug is less than or equal to 2.81 (per 100,000 people). State the null hypothesis and the alternative hypothesis.

$$H_0 : \mu \leq 2.81$$

$$H_1 : \mu > 2.81$$

As you can see, there are a few different types of null hypotheses (and therefore alternative hypotheses). When we encounter a claim, it is one of these three types:

(1)  $H_0 : \mu = c$  and  $H_1 : \mu \neq c$

(2)  $H_0 : \mu \leq c$  and  $H_1 : \mu > c$

(3)  $H_0 : \mu \geq c$  and  $H_1 : \mu < c$

(Note: The null hypothesis always includes the equal sign!)

Our goal when we test a claim will be to analyze the results of a sample and determine whether they allow us to reject the claim or not. For example, if we randomly sample 1,000 Americans and their mean income is \$208,912, it is likely that the government was wrong when they said the mean income of all Americans was only \$32,067. If however, our sample mean is \$31,950, then maybe their claim is correct. Again, our two possible results are that we (a) reject the claim or (b) do not reject the claim. We never are able to conclude that the claim is correct.

In order to make this decision, we have to decide how willing we are to be wrong. There are two types of errors we could make. If the null hypothesis is actually true, but we reject it, we have made a **type-I error**. If on the other hand, the null hypothesis is false, but we do not reject it, we have made a **type-II error**. This is summarized in the chart below.

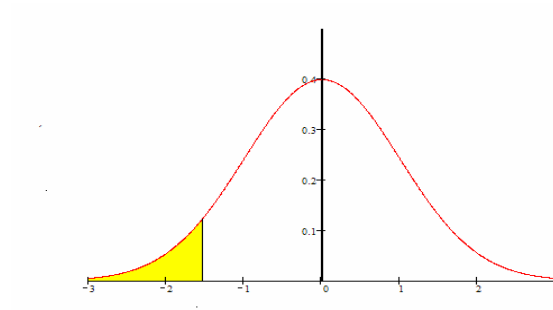
	$H_0$ is true	$H_0$ is false
We reject $H_0$	type-I error	Correct decision
We do not reject $H_0$	Correct decision	type-II error

The probability of making a type-I error is called the **level of significance** and is denoted by  $\alpha$ . (The probability of making a type-II error is usually denoted by  $\beta$ .) Typically, we use  $\alpha = 0.05$  or  $\alpha = 0.01$  (which means there is a 5% or a 1% chance of making a type-I error), but other values are possible. If  $\alpha$  is not specified, assume  $\alpha = 0.05$ .

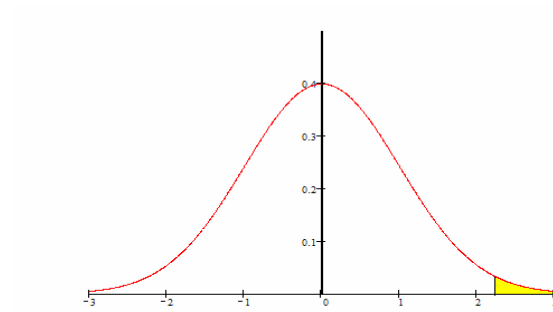
When making the decision on whether or not to reject the null hypothesis, we will consult the appropriate probability distribution (for means, this is the standard normal distribution). We will determine the **critical region** (which is the region in which we will reject the null hypothesis) and the **critical value** (the  $z$ -value that separates the critical region from the non-critical region). Then we'll compare the sample data to this value and make our decision. In the **one-tailed test** (either **right-tailed** or **left-tailed**), our critical region is at one far end of the standard normal distribution or the other. For example, if  $\alpha = 0.05$ , then the critical region would be the uppermost 5% (right-tailed) or the lowermost 5% (left-tailed). In the **two-tailed test**, our critical region will be equally divided at

both ends. So if  $\alpha = 0.05$ , then our critical region is the upper 2.5% *and* the lower 2.5%.

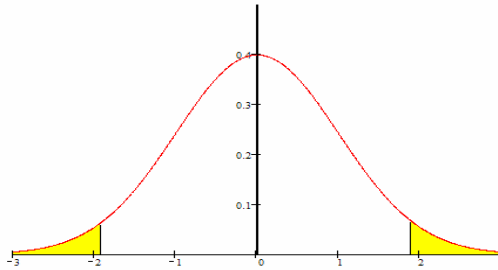
**Example 3:** Find the critical region and critical value for  $\alpha = 0.05$  with a left-tailed test.



**Example 4:** Find the critical region and critical value for  $\alpha = 0.01$  with a right-tailed test.



**Example 5:** Find the critical region and critical value for  $\alpha = 0.05$  with a two-tailed test.



**Example 6:** Find the critical region and critical value for  $\alpha = 0.03$  with a two-tailed test.

